Pion Form Factor With Twisted Mass QCD

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Motivation for Twisted Mass LQCD:

- Lattice QCD with Wilson fermions:

$$S_F^{LW} = a^4 \sum_x \bar{\psi}_p(x) \left[\sum_\mu \frac{1}{2} \gamma_\mu (\nabla_\mu^* + \nabla_\mu) + m_0 - a\frac{r}{2} \sum_\mu \nabla_\mu^* \nabla_\mu \right] \psi_p(x)$$

- $\mathcal{O}(a)$ errors

- Exceptional configurations in quenched calculations.

- Improvement with *clover* term and improved operators $\cdots$ **exceptional configurations** still exist.

Twisted Mass QCD and Exceptional Configurations:

– *R. Frezzotti, P. A. Grassi, S. Sint and P. Weisz, JHEP* **0108***, 058 (2001).*

– *R. Frezzotti, S. Sint and P. Weisz, JHEP* **0107***, 048 (2001).*

- In the continuum, the twisted mass action for *a degenerate quark doublet* is given by:

$$S_F[\psi,\bar{\psi}] = \int d^4x \bar{\psi}(D_\mu\gamma_\mu + m_q + i\mu_q\gamma_5\tau^3)\psi$$

- An axial transformation,

$$\psi' = e^{i\omega\gamma_5\tau^3/2}\psi, \quad \bar{\psi}' = \bar{\psi}e^{i\omega\gamma_5\tau^3/2}$$

leaves the form of the action invariant and rotates the masses as:

$$\begin{pmatrix} m'_q \\ \mu'_q \end{pmatrix} = \begin{bmatrix} \cos(\omega) & \sin(\omega) \\ -\sin(\omega) & \cos(\omega) \end{bmatrix} \begin{pmatrix} m_q \\ \mu_q \end{pmatrix}$$

- Quark mass:

$$M_q^2 = m_q^2 + \mu_q^2$$

- $\tan(\omega) = \frac{\mu_q}{m_q} \quad \rightarrow \quad \mu'_q = 0$ which is QCD.

- Composite field operators:

$$A'^a_\mu = \begin{cases} \cos(\omega) A^a_\mu + \epsilon^{3ab} \sin(\omega) V^b_\mu & (a = 1, 2), \\ A^3_\mu & (a = 3) \end{cases}$$

- Positive definite determinant of the Dirac operator
  $\rightarrow$ $\boxed{\textbf{No exceptional configurations}}$.

Twisted Mass and $\mathcal{O}(a)$ improvement:

– *R. Frezzotti and G. C. Rossi, Nucl. Phys. Proc. Suppl.* **129**, *880 (2004)*

– *R. Frezzotti and G. C. Rossi, arXiv:hep-lat/0306014.*

- Symmetries:

$$\begin{aligned} \mathcal{R}_5^{SP} &= \mathcal{R}_5 \times (r \to -r) \times (m_q \to -m_q) \\ \mathcal{R}_5 &\times \mathcal{D}_d \\ \mathcal{R}_5 &: \psi \to \gamma_5 \psi, \quad \bar{\psi} \to -\bar{\psi}\gamma_5 \\ \mathcal{D}_d &: U_\mu(x) \to U_\mu^\dagger(-x - a\hat{\mu}) \\ & \quad\ \psi(x) \to e^{3i\pi/2}\psi(-x) \end{aligned}$$

- Wilson averaging gives $\mathcal{O}(a)$ improved correlators:

$$\frac{1}{2}[\langle O\rangle_r^\omega + \langle O\rangle_{-r}^\omega] = \zeta_O(\omega, r)\,\langle O\rangle_{cont.} + \mathcal{O}(a^2)$$

- Wilson averaging is done with *tmQCD*.

- **Simplification at $\omega = \pm\frac{\pi}{2}$**

  - Matrix elements are either automatically improved or improved by averaging correlator with momenta $\vec{k}$ and $-\vec{k}$.

$$\begin{aligned} \left\langle n, \vec{k} | O | n', \vec{k'} \right\rangle^{\frac{\pi}{2}}_{r,m_q} &+ \eta_{nn'O} \left\langle n, -\vec{k} | O | n', -\vec{k'} \right\rangle^{\frac{\pi}{2}}_{r,m_q} \\ &= 2\zeta_O(r) \left\langle n, \vec{k} | O | n', \vec{k'} \right\rangle^{\frac{\pi}{2}}_{cont.} + \mathcal{O}(a^2) \end{aligned}$$

$$\eta_{nn'O} = \pm 1$$

$$E_n(\vec{k}, r, m_q) + E_n(-\vec{k}, r, m_q) = 2E_n^{cont.}(\vec{k}, m_q) + \mathcal{O}(a^2)$$

Details of Our Lattice Calculations:

- Lattice size: $16^3 \times 48$

- 100 gauge field configurations, quenched.

- $\boxed{\text{Matrix inversion by GMRES-DR}}$

  – *R. B. Morgan SIAM J. Sci. Comput.* **24**, *20 (2002).*

  – *R. B. Morgan and W. Wilcox, Nucl. Phys. Proc. Suppl.* **106**, *1067 (2002).*

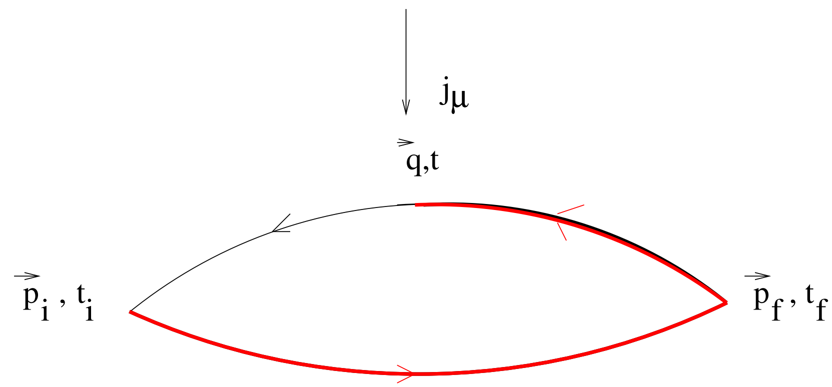
- $\beta = 6.0$, $\kappa = \kappa_c$, $\mu_q = 0.015, 0.030.$

  – *K. Jansen, A. Shindler, C. Urbach and I. Wetzorke, Phys. Lett. B* **586**, *432 (2004).*

The Pion Form Factor

- The pion form factor is of theoretical and experimental interest.
- No disconnected lattice diagrams.
- Transition from non-perturbative QCD to perturbative QCD occurs at smaller Q^2 than for baryons.
- Mainly described by Vector Meson Dominance.
- Experimental measurement at high Q^2 is underway at **JLab**.

- Various attempts to compute the pion form factor on the lattice:
  - Wilson (quenched)
    - *F. Bonnet, R. Edwards, G. Fleming, R. Lewis and D. Richards [LHP Collaboration], Nucl. Phys. Proc. Suppl.* **128***, 59 (2004), Nucl. Phys. Proc. Suppl.* **129***, 206 (2004).*
  - Clover (quenched)
    - *J. van der Heide, M. Lutterot, J. H. Koch and E. Laermann, Phys. Lett. B* **566***, 131 (2003).*
    - *J. van der Heide, J. H. Koch and E. Laermann, Phys. Rev. D* **69***, 094511 (2004).*
  - Domain Wall (quenched)
    - *Y. Nemoto [RBC Collaboration], Nucl. Phys. Proc. Suppl.* **129***, 299 (2004).*
  - Domain Wall (dynamical staggered configurations)
    - *G. Fleming [LHP Collaboration], to be presented in this conference.*



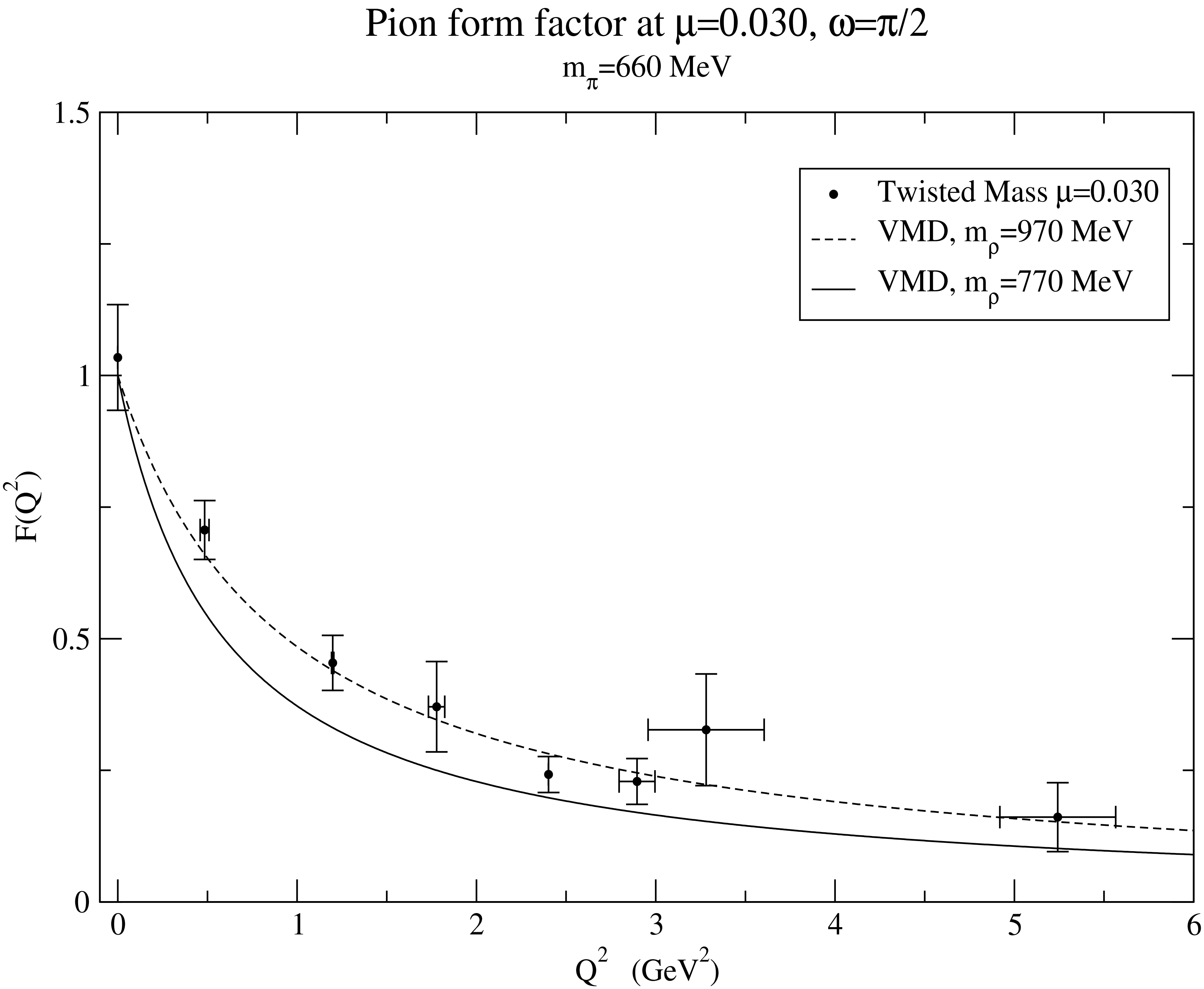
- It corresponds to the process (for example):

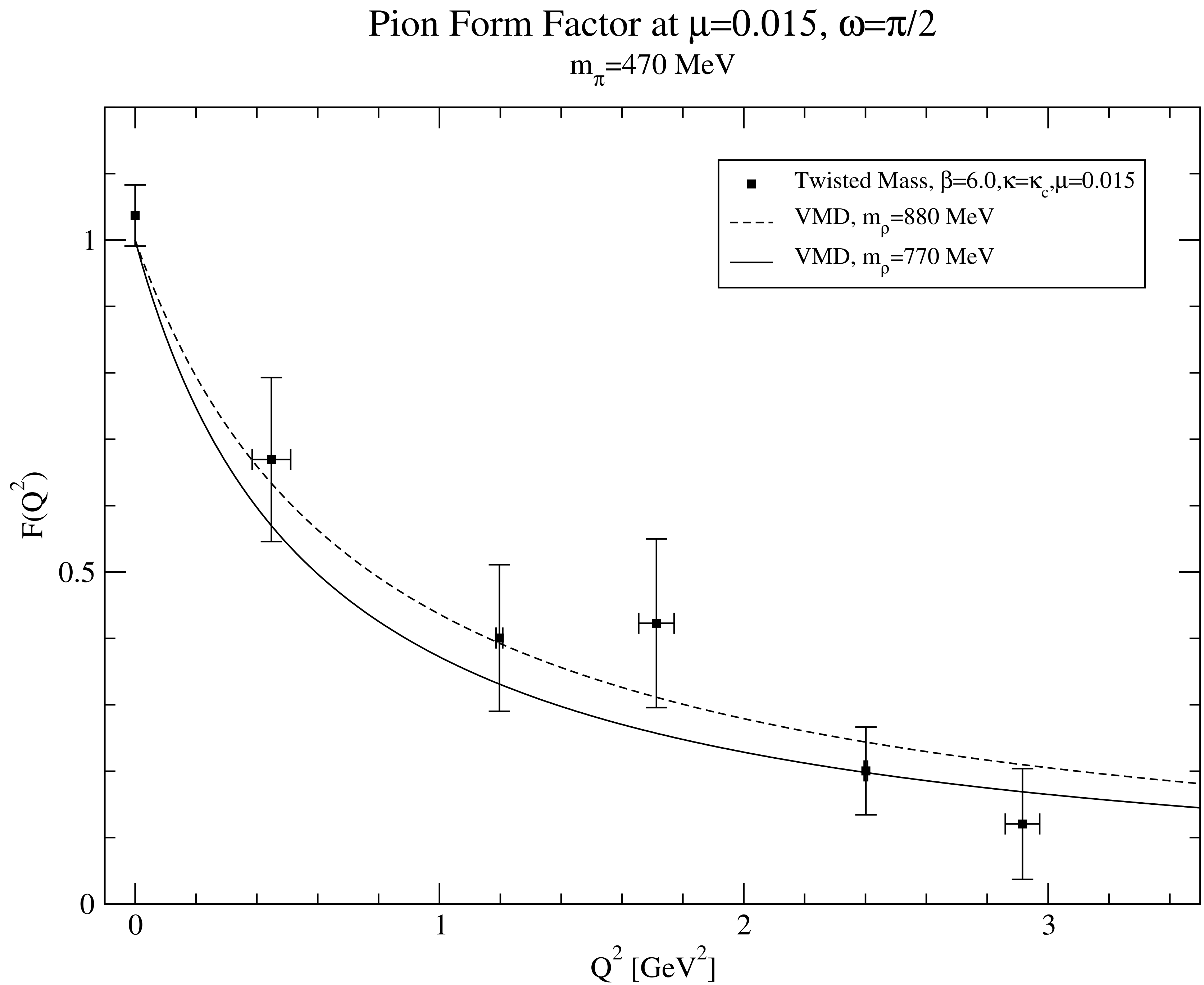
$$\pi(\vec{p_i}) \longrightarrow \pi(\vec{p_f})\gamma^*(\vec{q})$$

- The pion form factor $F(Q^2)$ is defined by:

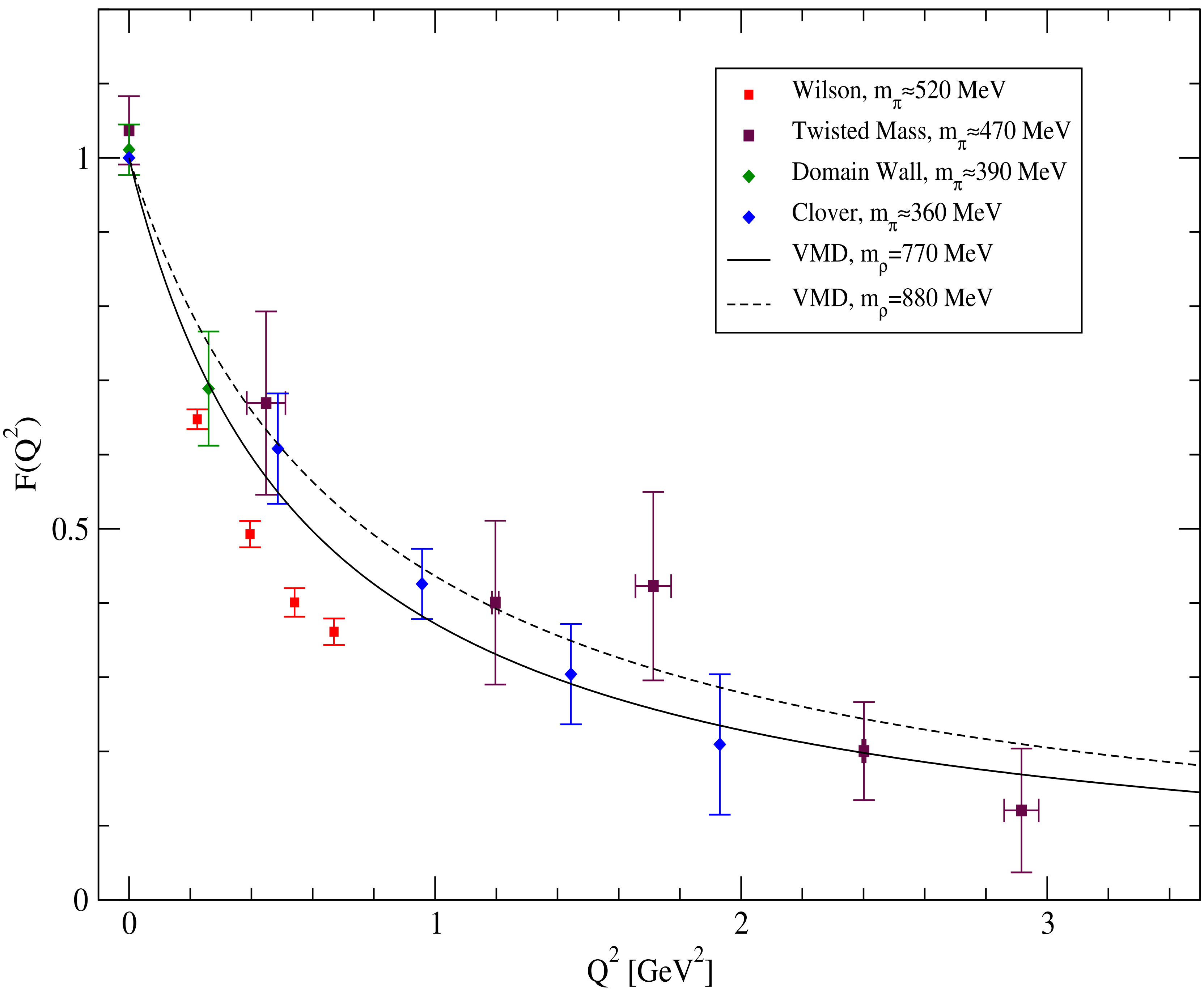
$$\begin{aligned} \left\langle \pi(p_f)|j_\mu(0)|\pi(p_i)\right\rangle_{cont.} &= F(Q^2)(p_i+p_f)_\mu \\ Q &= p_f - p_i \end{aligned}$$

- j_μ is a conserved vector current

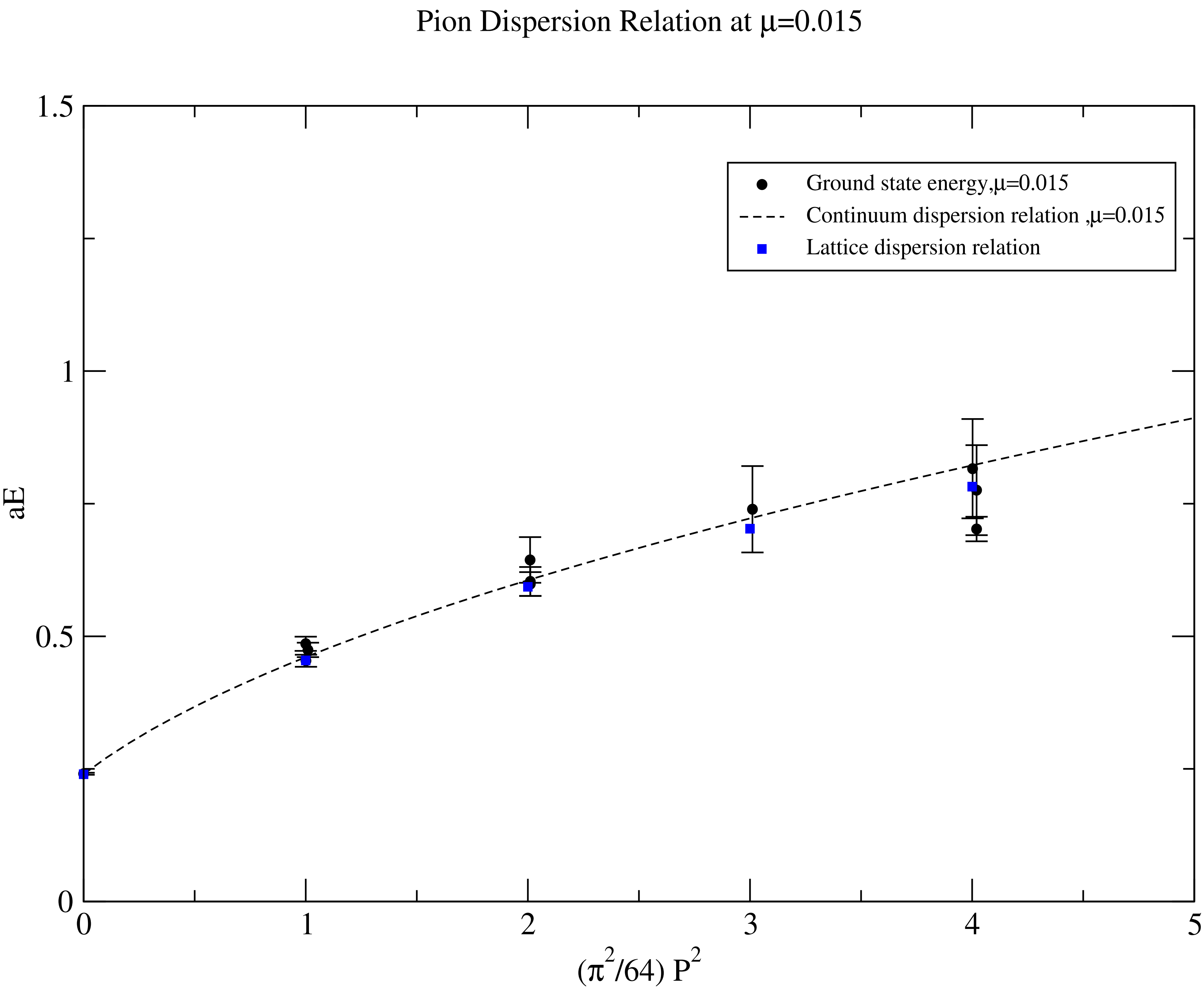

Pion form factor at μ=0.030, ω=π/2
m_π=660 MeV
Twisted Mass μ=0.030
VMD, m_ρ=970 MeV
VMD, m_ρ=770 MeV
$F(Q^2)$
Q^2 (GeV^2)
1.5
1
0.5
0
0
1
2
3
4
5
6



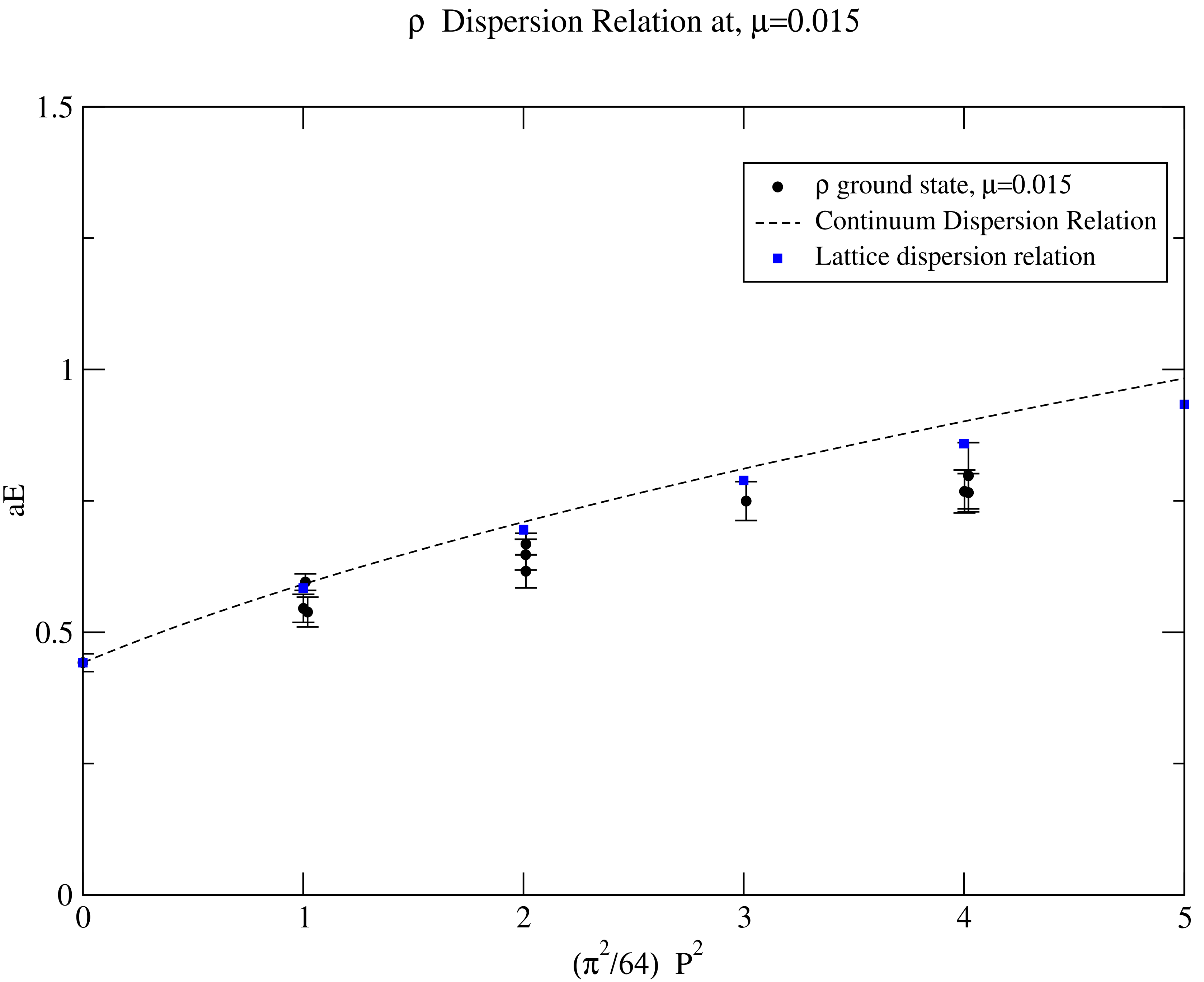
Pion Form Factor at μ=0.015, ω=π/2
m_π=470 MeV
Twisted Mass, β=6.0,κ=κ_c,μ=0.015
VMD, m_ρ=880 MeV
VMD, m_ρ=770 MeV
$F(Q^2)$
Q^2 [GeV^2]
0
0.5
1
0
1
2
3

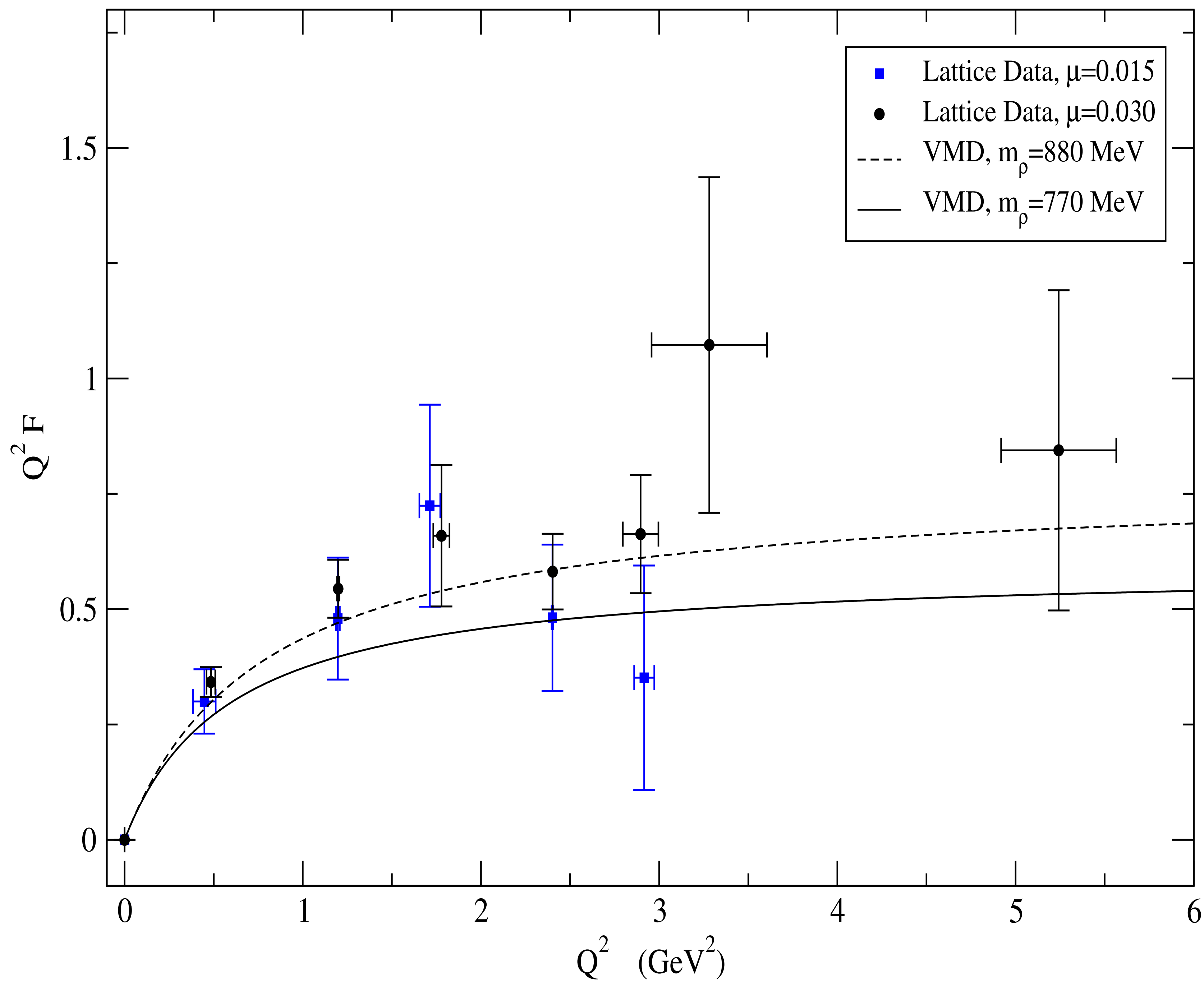


Wilson, m_π≈520 MeV
Twisted Mass, m_π≈470 MeV
Domain Wall, m_π≈390 MeV
Clover, m_π≈360 MeV
VMD, m_ρ=770 MeV
VMD, m_ρ=880 MeV
$F(Q^2)$
Q^2 [GeV^2]
0
0.5
1
0
1
2
3

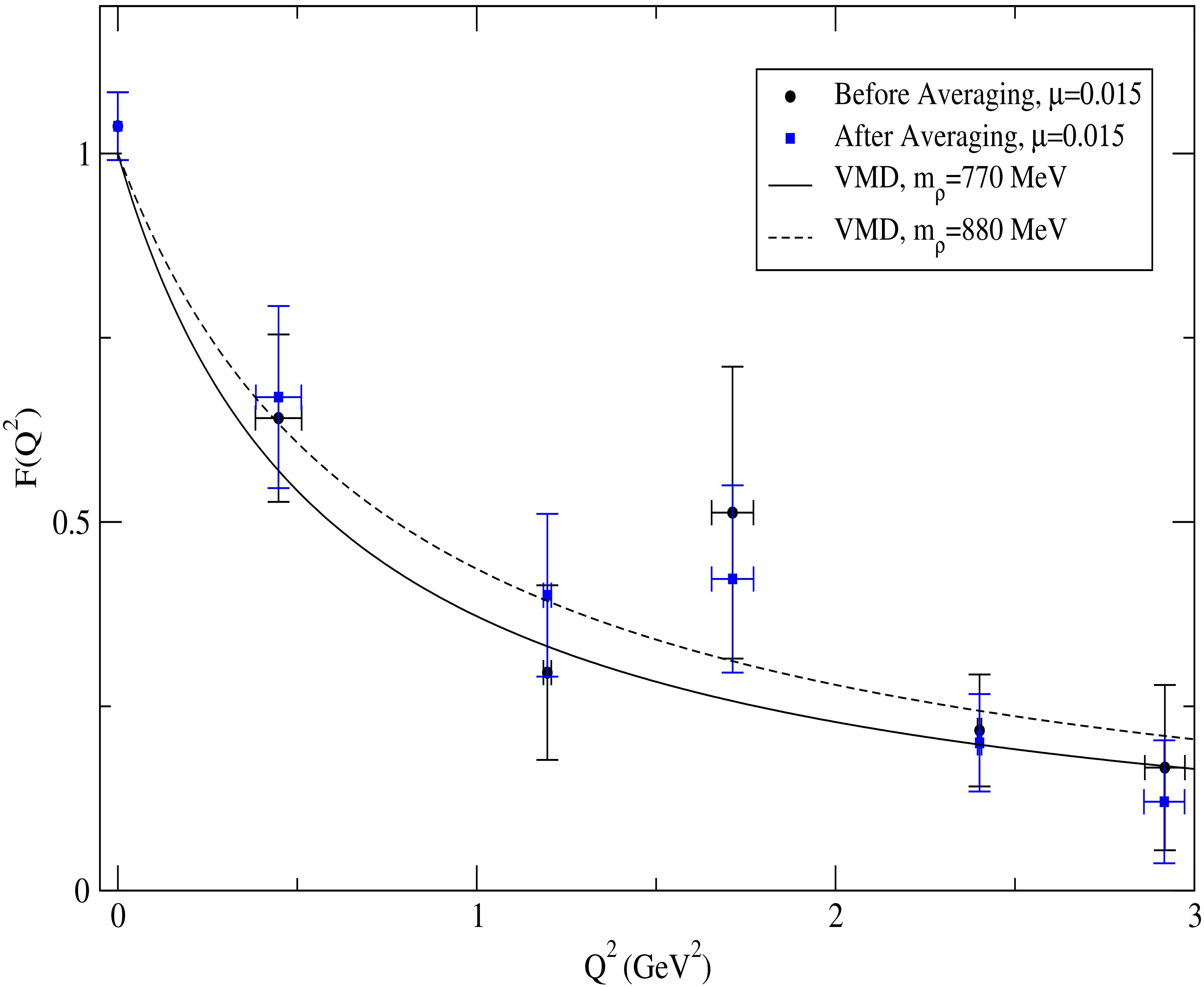


Pion Dispersion Relation at μ=0.015
Ground state energy,μ=0.015
Continuum dispersion relation ,μ=0.015
Lattice dispersion relation
aE
(π²/64) P²
0
0.5
1
1.5
0
1
2
3
4
5


ρ Dispersion Relation at, μ=0.015
ρ ground state, μ=0.015
Continuum Dispersion Relation
Lattice dispersion relation
aE
(π²/64) P²
0
0.5
1
1.5
0
1
2
3
4
5



Lattice Data, μ=0.015
Lattice Data, μ=0.030
VMD, m_ρ=880 MeV
VMD, m_ρ=770 MeV
$Q^2 F$
Q^2 (GeV^2)
0
0.5
1
1.5
0
1
2
3
4
5
6



Before Averaging, μ=0.015
After Averaging, μ=0.015
VMD, m_ρ=770 MeV
VMD, m_ρ=880 MeV
$F(Q^2)$
Q^2 (GeV^2)
0
0.5
1
0
1
2
3

Conclusions:

- $\boxed{\textit{tmQCD}}$ solves the problem of exceptional configurations.

- $\mathcal{O}(a)$ improvement could be obtained with a simple averaging procedure at $\omega = \pm\frac{\pi}{2}$ without the need for improvement terms.

- At $\omega = \pm\frac{\pi}{2}$ the results are improved as compared to Wilson even before averaging.

- $\boxed{\textit{GMRES-DR}}$ is an efficient matrix inverter for *tmQCD* calculations.

- The pion form factor was calculated to a high Q^2 value.